MATHEMATICAL DRIVING MODEL OF THREE PHASE INDUCTION MOTORS IN STATIONARY COORDINATE FRAME

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ABSTRACT: In this paper mathematical analysis for a design three-phase induction motor. Three phase AC induction motors is popular in manufacturing for many reasons. It is quite simple and minimum costs are favorable. It does not have brushes and requires minimum maintenance. In this paper a mathematical development to three-phase AC induction motor by using matrix form. The development applying on model of a real drive of 1.1 kW motor has been achieved. To analysis the axis, three-phase coordinate (A, B, C) to a two axis (β - α) as a (stationary part) representation. Through this drive conversion torque and speed can be controlled easily and control optimization induction motors. MATLAB/ SIMULINK is used to simulate the model of (IM) as tool and for study of characteristics of the motor.

Keywords: Induction motor; coordinate (α–β); MATLAB/ SIMULINK; mathematical analysis; stationary system; matrix form.

1- INTRODUCTION

The three-phase induction motors (squirrel cage) are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer DC motors when large speed variations are required. Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements.

In this paper, focus attention on the general principles of 3-phase induction motors\cite{1}. Modeling and simulation of a three-phase induction motor and its electronic drive are considered. The induction motor is modeled in stationary coordinates. The drive is a current regulated pulse-width modulated (PWM) inverter utilizing field oriented control (FOC). Simulation results in both motoring and generating modes are provided\cite{2}.

2-AC Induction Motors

2-1. Three-Phase Ac Induction Motor.

Induction motors derive their name from the way the rotor magnetic field is created. The rotating stator magnetic field induces currents in the short circuited rotor. These currents produce the rotor magnetic field, which interacts with the stator magnetic field, and produces torque, which is the useful mechanical output of the machine. The three phase squirrel cage AC induction motor is the most widely used motor. The bars forming the conductors along the rotor axis are connected by a thick metal ring at the ends, resulting in a short circuit as shown in Figure (1) \cite{3}. The sinusoidal stator phase currents fed in the stator coils create a
magnetic field rotating at the speed of the stator frequency (ωs) [4]. The changing field induces a current in the cage conductors, which results in the creation of a second magnetic field around the rotor wires[5]. As a consequence of the forces created by the interaction of these two fields, the rotor experiences a torque and starts rotating in the direction of the stator field. The stator accommodate 3-phase winding a,b,c. The turns in each winding are distributed so that a current in a stator winding produces an approximately sinusoidal-distributed flux density around the periphery of the air gap. When three currents that are sinusoidal varying in time, but displaced in phase by 120° from each other, flow through the three symmetrically-placed windings, a radially-directed air gap flux density is produced that is also sinusoidal distributed around the gap and rotates at an angular velocity equal to the angular frequency of the stator currents[6]. The most common type of induction motor has a squirrel cage rotor in which aluminium conductors or bars are cast into slots in the outer periphery of the rotor[7]. These conductors or bars are shorted together at both ends of the rotor by cast aluminium end rings, which also can be shaped to act as fans. In larger induction motors, copper or copper-alloy bars are used to fabricate the rotor cage winding [8].

As the sinusoidal-distributed flux density wave produced by the stator magnetizing currents sweeps past the rotor conductors, it generates a voltage in them. The result is a sinusoidal-distributed set of currents in the short-circuited rotor bars. Because of the low resistance of these shorted bars, only a small relative angular velocity between the angular velocity of the flux wave (ωs) and the mechanical angular velocity (ωm) of the two-pole rotor is required to produce the necessary rotor current. The relative angular velocity is called the slip velocity (ωslip) [9]. The interaction of the sinusoidal-distributed air gap flux density and induced rotor currents produces a torque on the rotor. The typical induction motor speed-torque characteristic is shown in Figure.(2) [10].

Squirrel-cage AC induction motors are popular for their simple construction, low cost per horsepower and low maintenance (they contain no brushes(brushless), as do DC motors). They are available in a wide range of power ratings. With field-oriented vector control methods, AC induction motors can fully replace standard DC motors, even in high-performance applications [11].


There are a lot of AC induction motor models; the model used for vector control design can be obtained by using the space vector theory. The 3-phase motor quantities (such as voltages, currents, magnetic flux, etc.) are expressed in terms of complex space vectors. Such model is valid for any instantaneous variation of voltage and current and adequately describes the performance of the machine under both steady-state and transient operation. Complex space vectors can be described using only two orthogonal axes. The motor can be considered as a 2-phase machine. The utilization of the 2-phase motor model reduces the number of equations and simplifies the control design [12].

2-2-1 Space Vector Definition

Assume that \(i_{sa}, i_{sb}, \) and \(i_{sc} \) are the instantaneous balanced 3-phase stator currents:

\[
i_{sa} + i_{sb} + i_{sc} = 0
\]

(1)

The stator current space vector can then be defined as follows:

\[
i_s = K \cdot (i_{sa} + a \cdot i_{sb} + a^2 \cdot i_{sc})
\]

(2)

Where: \(a \) and \(a^2 \) - the spatial operators, \(a = e^{j2\pi/3}, \quad a^2 = e^{j4\pi/3}.\)

\(k\) - The transformation constant and is chosen \(k=2/3.\)

The space vector defined by eq. (2) can be expressed utilizing the two-axis theory[13]. The real part of the space vector is equal to the instantaneous value of the direct-axis stator current component, \(i_{sa}\), and whose imaginary part is equal to the quadrature-axis stator current component, \(i_{sb}\). Thus, the stator current space vector in the stationary reference frame attached to the stator can be expressed as:
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\[ i_s = i_{sa} + j i_{sb} \]  \hspace{1cm} (3)

In symmetrical 3-phase machines, the direct and quadrature axis stator currents \( i_{sa}, i_{sb} \) are fictitious quadrature-phase (2-phase) current components, which are related to the actual 3-phase stator currents [14].

\[ i_{sa} = K \left( i_{sa} - \frac{1}{2} i_{sb} - \frac{1}{2} i_{sc} \right) \]  \hspace{1cm} (4)

\[ i_{sb} = K \cdot \frac{\sqrt{3}}{2} (i_{sb} - i_{sc}) \]  \hspace{1cm} (5)

The space vectors of other motor quantities (voltages, currents, magnetic fluxes, etc.) can be defined in the same way as the stator current space vector [14].

2-2-2 AC Induction Motor Model

The AC induction motor model is given by the space vector form of the voltage equations. The system model defined in the stationary \( \alpha, \beta \)-coordinate system attached to the stator is expressed by the following equations. Ideally, the motor model is symmetrical, with a linear magnetic circuit characteristic[15].

A. The stator voltage differential equations:

\[ V_{sa} = R_s \cdot i_{sa} + \frac{d}{dt} \psi_{sa} \]  \hspace{1cm} (6)

\[ V_{sb} = R_s \cdot i_{sb} + \frac{d}{dt} \psi_{sb} \]  \hspace{1cm} (7)

B. The rotor voltage differential equations:

\[ V_{ra} = 0 = R_r \cdot i_{ra} + \frac{d}{dt} \psi_{ra} + \omega \psi_{r \beta} \]  \hspace{1cm} (8)

\[ V_{rb} = 0 = R_r \cdot i_{rb} + \frac{d}{dt} \psi_{rb} + \omega \psi_{r \alpha} \]  \hspace{1cm} (9)

C. The stator and rotor flux linkages expressed in terms of the stator and rotor current space vectors:

\[ \psi_{sa} = L_s \cdot i_{sa} + L_m i_{ra} \]  \hspace{1cm} (10)

\[ \psi_{sb} = L_s \cdot i_{sb} + L_m i_{rb} \]  \hspace{1cm} (11)

\[ \psi_{ra} = L_r \cdot i_{ra} + L_m i_{sa} \]  \hspace{1cm} (12)

\[ \psi_{rb} = L_r \cdot i_{rb} + L_m i_{sb} \]  \hspace{1cm} (13)

D. Electromagnetic torque expressed by utilizing space vector quantities.

\[ t_e = \frac{3}{2} P_p \cdot \left( \psi_{sa} \cdot i_{sb} - \psi_{sb} \cdot i_{sa} \right) \]  \hspace{1cm} (14)

Besides the stationary reference frame attached to the stator, motor model voltage space vector equations can be formulated in a general reference frame, which rotates at a general speed, \( \omega_e \). If a general reference frame, with direct and quadrature axes \( \alpha, \beta \) rotating at a general instantaneous speed \( \omega_e = d \theta_e / dt \) is used, as shown in Fig. (4), where \( \theta_e \) is the angle between the direct axis of the stationary reference frame (\( \alpha \)) attached to the stator and the real axis (\( x \)) of the general reference frame, then the following equation defines the stator current space vector in general reference frame[16]:

\[ i_{sg} = i_s e^{-j \theta_e} = i_{sa} + j i_{sy} \]  \hspace{1cm} (15)
The stator voltage and flux-linkage space vectors can be similarly obtained in the general reference frame. Similar considerations hold for the space vectors of the rotor voltages, currents and flux linkages. The real axis (\(r\alpha\)) of the reference frame attached to the rotor is displaced from the direct axis of the stator reference frame by the rotor angle, \(\theta_r\). As shown, the angle between the real axis (\(x\)) of the general reference frame and the real axis of the reference frame rotating with the rotor (\(r\alpha\)) is \(\theta_x-\theta_r\). In the general reference frame, the space vector of the rotor currents can be expressed as:

\[
i_{r\alpha} = i_r e^{-j(\theta_r-\theta_x)} = i_{rx} + j_{ry}
\]

(16)

Where: \(i_r\) - The space vector of the rotor current in the rotor reference frame.

The space vectors of the rotor voltages and rotor flux linkages in the general reference frame can be expressed similarly. The motor model voltage equations in the general reference frame can be expressed by using the transformations of the motor quantities from one reference frame to the general reference frame introduced. The AC induction motor model is often used in vector control algorithms. The aim of vector control is to implement control schemes which produce high-dynamic performance and are similar to those used to control DC machines. To achieve this, the reference frames may be aligned with the stator flux-linkage space vector, the rotor flux-linkage space vector or the magnetizing space vector. The most popular reference frame is the reference frame attached to the rotor flux linkage space vector with direct axis (\(d\)) and quadrature axis (\(q\)). After transformation into \(d\)-\(q\) coordinates the motor model follows [17].

\[
V_{sd} = R_s \cdot i_{sd} + \frac{d}{dt}\Psi_{sd} - \omega \Psi_{sq}
\]

(17)

\[
V_{sq} = R_s \cdot i_{sq} + \frac{d}{dt}\Psi_{sq} - \omega \Psi_{sd}
\]

(18)

\[
V_{rd} = 0 = R_r \cdot i_{rd} + \frac{d}{dt}\Psi_{rd} - (\omega_s - \omega) \cdot \Psi_{rq}
\]

(19)

\[
V_{rq} = 0 = R_r \cdot i_{rq} + \frac{d}{dt}\Psi_{rq} - (\omega_s - \omega) \cdot \Psi_{rd}
\]

(20)

\[
\Psi_{sd} = L_s \cdot i_{sd} + L_m \cdot i_{rd}
\]

(21)

\[
\Psi_{sq} = L_s \cdot i_{sq} + L_m \cdot i_{rq}
\]

(22)

\[
\Psi_{rd} = L_r \cdot i_{rd} + L_m \cdot i_{sd}
\]

(23)

\[
\Psi_{rq} = L_r \cdot i_{rq} + L_m \cdot i_{sq}
\]

(24)

\[
t_e = \frac{3}{2} P_p \left( \Psi_{sd} \cdot i_{sq} - \Psi_{sq} \cdot i_{sd} \right)
\]

(25)

The diagram in Fig. (5) displays \(d\)-\(q\) reference frame and the relationships between the stator voltage (\(U_s\)), stator current (\(I_s\)), and the rotor, stator and magnetizing flux (\(\Psi_r\), \(\Psi_s\), \(\Psi_m\)). The rotor magnetizing flux space-vector is aligned to the \(d\)-axis of the \(d\)-\(q\) reference frame.

In the steady state the AC induction motor model can be represented with the help of an equivalent steady state circuit, which is shown in Fig. (6).

In this paper we implement the scheme model of the induction motor in accordance with their option, it is assumed that the angular frequency of the supply voltage \(\omega_{leq} = 314\) rad/s, and the peak value of the phase voltage on the stator \(V_1 = 311\) V. The original system of equations (26) is used for the projections of the space vector in which the vectors are stored in a fixed coordinate system \(\alpha-\beta\) and which rotate with an angular velocity relative to the fixed coordinate system.
\[
\begin{align*}
\vec{U}_{1a-\beta} & = R_1 \vec{i}_{1a-\beta} + \frac{d}{dt} \vec{\psi}_{1a-\beta}; \\
\vec{U}_{2a-\beta} & = R_2 \vec{i}_{2a-\beta} + \frac{d}{dt} \vec{\psi}_{2a-\beta} - j\omega \vec{\psi}_{2a-\beta}; \\
\vec{\psi}_{1a-\beta} & = L_1 \vec{i}_{1a-\beta} + L_m \vec{i}_{2d-\beta}; \\
\vec{\psi}_{2a-\beta} & = L_2 \vec{i}_{1a-\beta} + L_m \vec{i}_{2d-\beta}.
\end{align*}
\]
(26)

Where \( R_1 \)-phase resistance of the stator winding; \( R_2 \) – phase resistance of rotor winding referred to the stator; \( M_c \) static load torque; \( \vec{\psi}_{1a-\beta} \) - vector of the stator flux linkage; \( \vec{\psi}_{2a-\beta} \) - vector of the rotor flux linkage; \( J \) - moment of inertia reduced to the motor; \( V_{1a-\beta} \) stator voltage vector; \( V_{2a-\beta} \) - vector rotor voltage; \( \omega \) - nominal angular velocity; \( \omega_r \) - speed of the rotor; \( L_1 \) - total inductance phase stator winding; \( L_2 \) - total inductance phase rotor winding; \( L_m \) - magnetizing inductance circuit. This mathematical can be represented in the following matrix form. Equation (27) the stator and rotor.

\[
V_3 = R_3 \cdot i_3 + \frac{d}{dt} \vec{\psi}_3 - p_n \cdot C \cdot \vec{\psi}_3
\]
(27)

Where the column matrices of voltages, currents and flux linkages are:

\[
\begin{bmatrix}
V_{1s} \\
V_{1f} \\
V_{2s} \\
V_{2f}
\end{bmatrix} =
\begin{bmatrix}
\psi_{1s} \\
\psi_{1f} \\
\psi_{2s} \\
\psi_{2f}
\end{bmatrix} =
\begin{bmatrix}
I_{1s} \\
I_{1f} \\
I_{2s} \\
I_{2f}
\end{bmatrix}
\]
(28)

\[R_3\] resistance matrix and the matrix \( C \), flux vector-turning on, have the form

\[
R_3 =
\begin{bmatrix}
R_1 & 0 & 0 & 0 \\
0 & R_1 & 0 & 0 \\
0 & 0 & R_2 & 0 \\
0 & 0 & 0 & R_2
\end{bmatrix}
\]
\[C =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]
(29)

Flux linkage and torque of the motor connected to the current expressions

\[
\psi_3 = L_3 \cdot I_3,
\]
(30)

\[
M = (G \cdot \Psi_3)^T \cdot (F \cdot I_3)
\]
(31)

The matrix of induction \( L_3 \), \( F \) and \( G \) are:

\[
L_3 =
\begin{bmatrix}
L_1 & 0 & L_m & 0 \\
0 & L_1 & 0 & L_m \\
L_m & 0 & L_2 & 0 \\
0 & L_m & 0 & L_2
\end{bmatrix};
F =
\begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix};
G =
\begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
(32)

The values for the model are: the magnitude of the voltage and rotor stator \( V_3 \), and the moment resistance \( M_r \).[18].

\[
V_3 =
\begin{bmatrix}
V_{1s} \\
V_{1f} \\
0 \\
0
\end{bmatrix}
\]
(33)
3-MODELING OF A 3-PHASE INDUCTION MOTOR

3-1 Space Vector Definition

3-1-1 Model of an Induction motor in a Stationary Coordinate System A-B

By use specification data, calculate the values of the parameters in the mathematical description of the induction motor.

3-1-2 COMPUTER MODEL OF A COORDINATE SYSTEM A-B

The voltage is applied in a matrix form in the stator flux linkage model \((P_{a1})\) stator current \(I_1\) in vector form [2].

Stator voltage, \(V_1 = \begin{bmatrix} V_{a1} & V_{b1} \end{bmatrix}\);

Stator current, \(I_1 = \begin{bmatrix} I_{a1} & I_{b1} \end{bmatrix}\);

Stator flux, \(\psi_1 = \begin{bmatrix} \psi_{a1} & \psi_{b1} \end{bmatrix}\); \(\psi\) in the circuit calculated rotor flux linkage \(\Psi\) and the current \(I_2\) in a matrix form.

Rotor current, \(I_2 = \begin{bmatrix} I_{a2} & I_{b2} \end{bmatrix}\);

Rotor flux linkage, \(\psi_2 = \begin{bmatrix} \psi_{a2} & \psi_{b2} \end{bmatrix}\);

To rotate the vector used matrix B.

\(B = \begin{bmatrix} 0 & 1 \\ 1 & -0 \end{bmatrix}\)

3-1-3 Modeling of the Motor Three-Phase Into A Two-Phase

The design to subsystem in figure (9) to test motor to input voltage (ABC) sinusoidal. First, to solve these problems, we consider two procedures. In order to simplify the model used by the coordinate transformation method. It is used to convert three-phase stationary variables \((a, b, c)\) in a two-phase stationary frame \((\alpha, \beta)\) [12]. Converting three-phase voltage to the desired two-phase model is carried out by the expressions.

\[
V_{a1} = \frac{2}{3}(V_a - \frac{V_b + V_c}{2}) \quad (34)
\]

\[
V_{b1} = \frac{2}{3} \sqrt{3} (\frac{V_b + V_c}{2}) \quad (35)
\]

Converting the two-phase fixed coordinate system \((\alpha-\beta)\) coordinate system in three-phase (A, B, C), using the equation:

\[
I_{1A} = I_{a1} \quad (36)
\]

\[
I_{1B} = -\frac{I_{a1} + \sqrt{3}}{2} I_{b1} \quad (37)
\]

\[
I_{1C} = -\frac{I_{a1} - \sqrt{3}}{2} I_{b1} \quad (38)
\]

These equations correspond to the model as shown in Fig. (10)

Based on the mathematical description of computer model developed in mat lab package shown in Fig. (11).

4- SIMULATION RESULTS

The figure (12) shows the represented of the three phase voltage (ABC) rms value 480volt, while the phase angle between then 120°.

The figure (13) shows the voltage of \((\alpha-\beta)\) after converted three phase voltage \((ABC)\) to two phase voltage \((\alpha - \beta)\). voltage of \((\alpha-\beta)\) for the stator motor 480 volt and the rotor voltage equal to zero.

The figure (14) shows the three phase current motor (ABC) with time. With value 24 Ambers. And get stable with 4A with time 0.2 sec.
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The figure (15) shows the (Torque and Speed) with change in the time setting (2sec) the torque increase and we observe the value of speed decrease in the 1 sec. the transient value for torque with disappear with time 0.2 sec. with load torque increased.

The figure (16) represented Two axis current rotor and stator($\alpha - \beta$).

The figure (17) represented the Relationship between three phase Currents (ABC) and (Is) with time.

5- CONCLUSION

In this paper, development of a mathematical model for 3 phases induction motor (squirrel cage) drive system by using MATLAB/SIMULINK. This development has been achieved by using matrix form, and applied on model of a real drive of 1.1 kW motor. Simulation result shows that the model was developed to reduce the cost and increases the overall efficiency. To analysis the axis, the conversion from three-phase coordinate (A, B, C) to a two axis ($\beta - \alpha$) as a (stationary part) represented and achieved. This converted was a control method on the torque and speed for the motor, and it was a good selected and easily way for control. The results show a good agreement to the theoretical background of the drive system.

REFERENCES

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Figure 1. 3-Phase AC Induction Motor

Figure 2. AC Induction Motor Speed-Torque Characteristic

Figure 3. Stator Current Space Vector and Its Projection
Figure 4. Phases diagram to phase current stationary form

Figure 5. Induction Motor Space-Vector Diagram

Figure 6. Equivalent circuit Induction Motor

Figure 7. Three phase input voltage sinusoidal

Figure 8. Phase difference between (A, B, C)
Figure 9. System convert three-phase phase voltage (abc) in (α β).

Figure 10. System for converting two phase current (α β) in the phase (ABC).

Figure 11. Model of 3 phase IM by simulink

Figure 12. Relationship between Three input voltage (ABC) with Time.

Figure 13. Relationship between two phase voltage stationary form (α – β) and time.
Figure 14. Three phase current motor (ABC) with time.

Figure 15. The result torque and Rotor speed vs time

Figure 16. Two axis current rotor and stator (α – β)